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Power fluctuations in a closed turbulent shear flow

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We study experimentally the power consumption P of a confined turbulent flow at constant Reynolds number Re. We analyze in details its temporal dynamics and statistical properties, in a setup that covers two decades in Reynolds numbers. We show that nontrivial power fluctuations occur over a wide range of amplitudes and that they involve coherent fluid motions over the entire system size. As a result, the power fluctuations do not result from averaging of independent subsystems and its probability density function $\Pi(P)$ is strongly non-Gaussian. The shape of $\Pi(P)$ is Reynolds number independant and we show that the relative intensity of fluctuations decreases very slowly as Re increases. These results are discussed in terms of an analaogy with critical phenomena. [S1063-651X(99)50709-5]

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In the study of turbulence, much work has been devoted to the description of small scale properties. Experimental data and models have been concerned with the scaling of local quantities such as the velocity increment $\delta u(r)\equiv\langle u(x+r)-u(x)\rangle_x$. It has then been shown that a key feature is the statistics of the energy transfer rate from the integral to the dissipative scale [1,2]. However, in the developement of a statistical description of turbulence it will be necessary to link these observations to the behavior of global quantities, as is done in statistical thermodynamics and dynamical systems driven far from equilibrium [3]. In particular, very little is known about the global power consumption P of a fully turbulent flow. The standard phenomenology assumes that P is constant in time, or that it has very small Gaussian fluctuations. Recent studies, both experimental [4] and numerical [5,6] have shown that this is not always so, at least for finite Reynolds numbers.

In this paper we present experimental measurements on the temporal fluctuations $P(t)$ in the case of a closed flow. We show that the nontrivial fluctuations can be attributed to the intermittent development of coherent structures involving the entire flow. Such events occur over a wide range of amplitudes and, for fixed amplitude, obey Poisson statistics. They are responsible for the asymmetric, strongly non-Gaussian, probablity distribution function $\Pi(P)$ which remains unchanged over two decades in Reynolds number. We discuss these results in the context of an analogy with a critical system, proposed in an earlier work [7].

The experimental setup and measurement techniques are those described in [4]. The flow belongs to the von Kármán geometry [8]. It is produced in the gap between two counter rotating disks of radius $R=12$ cm. The fluid is confined in a cylindrical vessel of height $H=41$ cm and radius $R'=24$ cm. This arrangement is known to produce an intense turbulence in a compact region of space [9]. The disks are driven by two dc motors whose rotation rate Ω is adjustable in the range [15,45] Hz and kept constant using a feedback loop. As a result, for a given value of Ω, the integral Reynolds number of the flow $\mathrm{Re}=R^2\Omega/\nu$ is constant. The power consumption of the flow is measured from the voltage and current consumption of the motors driving the disks. When ohmic and friction losses are discounted (see [4]) only the power P_1, P_2 injected into the flow by the disks (1) and (2) remain. Our study is devoted to P_1 and P_2, and the total injected power $P=P_1+P_2$. To achieve an extended range of Reynolds numbers, we note that all gases have roughly the same dynamical viscosity, so that their kinematic viscosity changes in inverse proportion to their atomic weights. We thus use helium, air, and carbon dioxide as working fluids. Then, Re in the flow varies from $\mathrm{Re}_{\mathrm{min}}=15\,100$ in helium at $\Omega=15$ Hz to $\mathrm{Re}_{\mathrm{max}}=500\,600$ in CO_2 at $\Omega=45$ Hz. In all cases we have verified that the flow is fully turbulent using local hot-wire anemometry, the spectra display a "$-5/3$" scaling range and the structure function exponents are in agreement with those usually reported in traditional turbulent setups [10].

Figure 1(a) shows an example of the time variations of the power injected by each disk, P_1 and P_2, and of the total power $P=P_1+P_2$, in air at $\Omega=45$ Hz. In each time series one observes asymmetric fluctuations about the mean with the occurance of power drops. Two noteworthy features are first that power drops occur over a wide range of magnitude and second that practically all events are felt simultaneoulsy at the two motors. As a result $P_1(t)$ and $P_2(t)$ are strongly correlated. This is illustrated in Fig. 1(b), where we plot the autocorrelation functions $\chi(P_1,P_1;\tau)$ (□ symbols) and $\chi(P_2,P_2;\tau)$ (○ symbols). One observes that the fluctuations of power injection at each disk are correlated at all times up to about 20 integral time scales Ω^{-1}. We attribute this to the occurrence of coherent fluid motion that can correlate the entire flow over many periods of rotation of the disks. Indeed, calculation of the cross-correlation function, $\chi(P_1,P_2;\tau)$ (* symbols), shows that it has a maximum equal to 0.8 for a time lag $\tau=0$ [see inset in Fig. 1(b)]. That is, fluctuations over a wide range of amplitudes all occur on length scales up to the order of the flow size.

As a result we cannot expect the power fluctuations to obey Gaussian statistics. The probability distribution func-

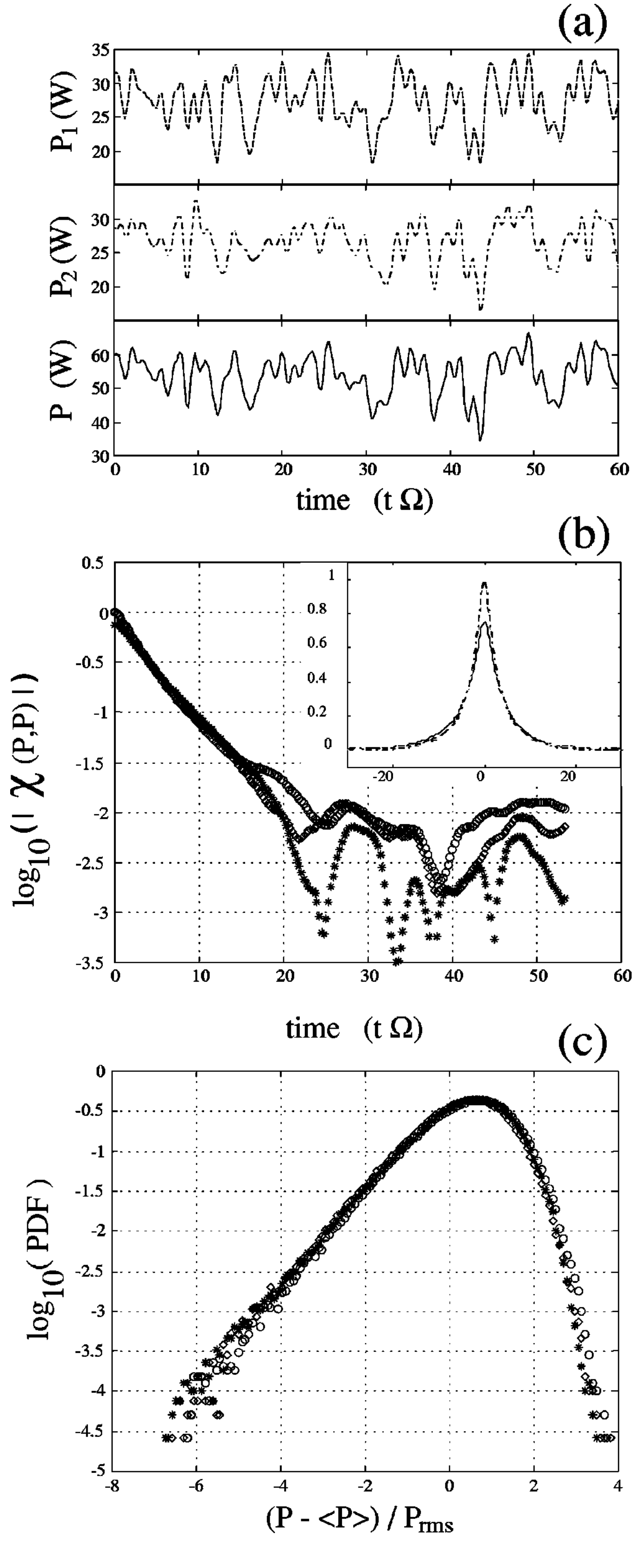

FIG. 1. (a) Variation with time (measured in units of the disks' rotation period) of the injected power (measured in Watts for each disk and total); (b) cross-correlation (*) and auto-correlation (○,□) functions in time of P_1 and P_2 (the inset display is in linear coordinates; the cross-correlation function $\chi(P_1,P_2;\tau)$ is plotted using a solid line); (c) PDF of power fluctuations in disk 1 (○), disk 2 (□), and total ones (*). Measurements performed in air at $\Omega = 45$ Hz (Re$=318{,}600$).

tions (PDFs) for P_1, P_2 and the total injected power $P = P_1 + P_2$ are given in Fig. 1(c). After normalization, the three curves are identical with an exponential tail towards the low values that extends to at least -6 standard deviations in variation. We note that a similar PDF has been observed for the fluctuations in the production term (Reynolds stress) in a numerical simulation of turbulence in the presence of a large scale shear [5]. We also remark for interest that such a variation from Gaussian statistics can be of practical importance; for example, the probability of observing a negative fluctuation of 6 standard deviations is 10^{-4} here, while it would be 10^{-8} for a Gaussian process.

The existence of the exponential tail is therefore a consequence of the occurrence of power drops caused by the formation of ''coherent structures,'' with a characteristic size of the order of the flow integral length scale. Here, ''coherent'' refers to large scale extension, in space and time, of the flow modifications. Indeed, if one averages out in a synchronous manner the power drops of amplitude larger than, for example, 3.5 standard deviations [11], one obtains the pattern shown in Fig. 2(a). The averaged power drop lasts for about 20 periods of rotation Ω^{-1}, in agreement with the characteristic time observed for the correlation function in Fig. 1(b). We have also computed the waiting time Δt between such events. The mean value $\overline{\Delta t}$ is about 400 times the period of rotation Ω^{-1} of the disks. Recalling that the power correlation functions are null for times greater than about $20\,\Omega^{-1}$, we observe that these strong power drops occur as decorrelated events. They obey Poisson statistics, as illustrated by the exponential decay of the PDFs of waiting times shown in Fig. 2(b). We remark here that the choice of 3.5 standard deviations is arbitrary. Other choices produce similar objects of different depth and with lifetime roughly in proportion. We propose therefore that the non-Gaussian tail of the PDF comes from the fact that a fluctuation of any magnitude can be produced by a single event.

We now discuss the behavior of the power fluctuations with the Reynolds number. In Fig. 3(a) we plot the PDFs of power fluctuations, normalized to the rms intensity, when the data span almost two orders of magnitude in Re. The measurements all collapse onto a single curve; the broadening in the wings is attributed to statistical uncertainties and no systematic trend can be detected. This confirms and extends our former observation [4] that the PDFs have a ''universal'' form, independent of Reynolds number. There is no evidence that, as Re increases, the power fluctuations will eventually reduce to Gaussian fluctuations.

The behavior of the relative intensity of the power fluctuations is also of interest. We first note that the mean value of the power consumption is set by the rate at which kinetic energy is supplied at large scales [4,12]. Dimensional analysis yields $\overline{P} = K\rho L^2 U^3$, where the constant K depends solely on the flow geometry. With our experimental setup, one has $\overline{P} = KR^5\Omega^3$, with $K = 59 \pm 0.5$. As for the value of the rms fluctuations, several arguments can be made. Based on standard turbulence phenomenology [1] one would assume that $P_{rms}/\overline{P}$ should decrease with increasing Re because of the increasing weight of small scale motion which can smooth out more and more efficiently the fluctuations in P. The variation measured in our experiment is plotted in Fig. 3(b).

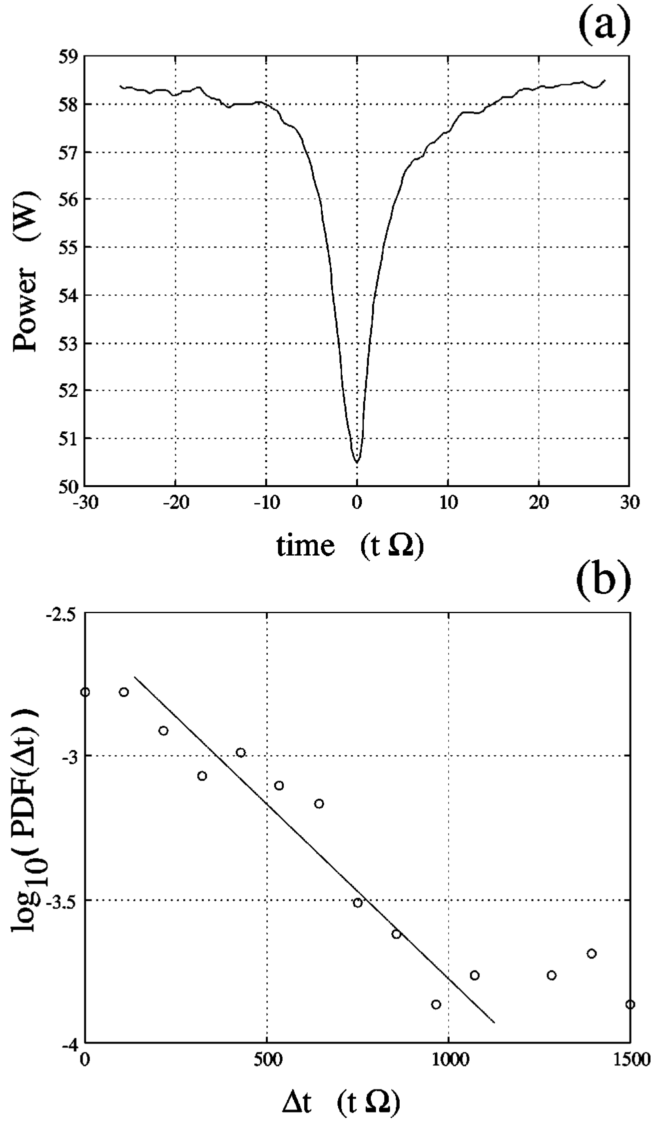

FIG. 2. (a) Synchronous average pattern of the power drops larger than 3.5 standard deviations below the mean. (b) PDF of waiting times between such events. Measurements performed in air at $\Omega=45$ Hz (Re$=318\,600$). Time is given in units of the disks rotation period.

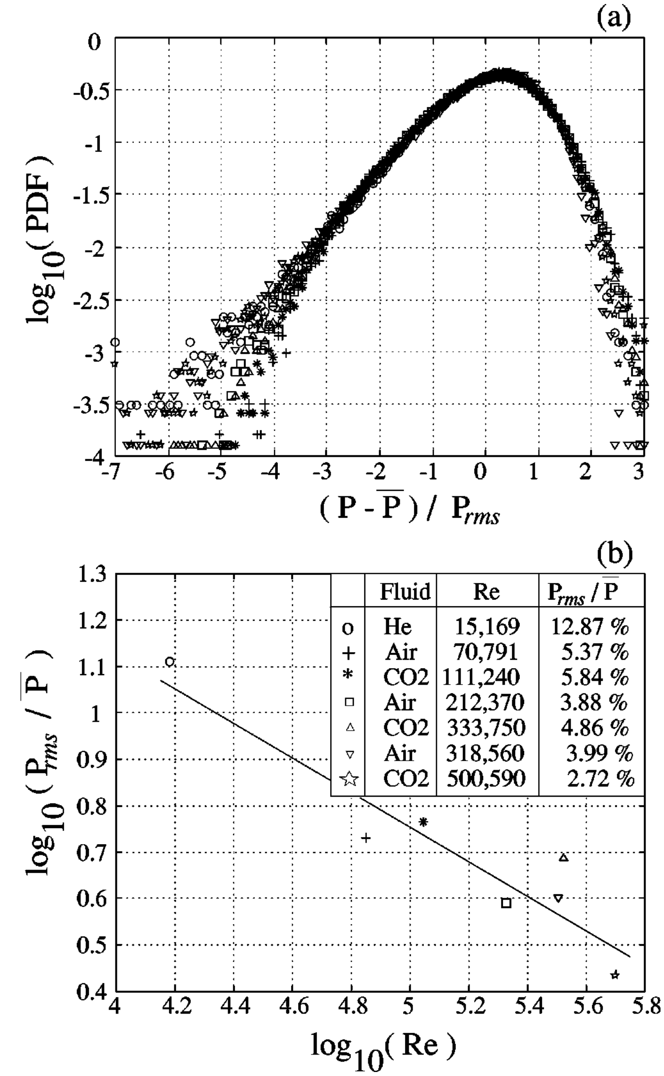

FIG. 3. (a) PDF of normalized power fluctuations; symbols as in the inset of (b). (b) Evolution with the Reynolds number of the relative intensity of power fluctuations.

We observe a slow decrease of $P_{rms}/\overline{P}$ with Re. A power law fit yields $\mathrm{Re}^{-\alpha}$ with $\alpha=0.33\pm0.05$, but logarithmic behavior cannot be ruled out.

These results can all be put into the context of an analogy with the order parameter fluctuations of a finite size critical system [7]. In both systems many length scales are important; in the turbulent flow energy cascades downward from the integral length L to the disipative length η. At a critical point fluctuations grow from the microscopic scale a upwards as the correlation length diverges. In a finite system this divergence is interrupted by the integral length L. In this case a scaling hypothesis [13] predicts that the PDF is universal with respect to system size. In the case of turbulence the equivalent result is that $\Pi(P)$ is independent of Reynolds number, a result which we observe here over two orders of magnitude.

With regard to the ratio $P_{rms}/\overline{P}$, a naive prediction for an uncorrelated system would be that $P_{rms}/\overline{P}$ varies in inverse proportion to the square root of the number of degrees of freedom [14], $N\sim\mathrm{Re}^{9/4}$, of the flow; that is, $P_{rms}/\overline{P}$ would vary as $\mathrm{Re}^{-9/8}$. On the other hand, a detailed comparison with a standard magnetic system with two independent critical exponents leads to the prediction that $P_{rms}/\overline{P}$ should be independent of N. The data, showing dependence on N, but with decay slower than $N^{-1/2}$ is consistent with a third independent exponent, which occurs, for example, in magnetic systems in the presence of a random field [15].

In conclusion, we have established that the strongly non-Gaussian fluctuations of the PDF for power consumption are due to events of coherent fluid motion spanning the entire system. We confirm that the PDF is a universal function of Reynolds number and we show that the relative magnitude of the fluctations $P_{rms}/\overline{P}$ decreases more slowly with Reynolds number than one would expect for a system of uncorrelated degrees of freedom. Our study of correlations between power drops at the two disks located at two extremes of the flow shows that the coherent structures occur over a wide range of amplitudes. The analogy with fluctuations in a finite size critical system [7] suggests that the coherent structures result from the strong coupling between modes over a large range of scales. This motivates further work to identify coherent structures in turbulent flows and to study their composition and their dynamics.

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[1] A. S. Monin and A. M. Yaglom, *Statistical Fluid Mechanics* (MIT Press, Cambridge, MA, 1971).

[2] U. Frisch, *Turbulence* (Cambridge University Press, Cambridge, England, 1995).

[3] G. Gallavotti and E.G.D. Cohen, Phys. Rev. Lett. **74**, 2694 (1995).

[4] R. Labbé, J.-F. Pinton, and S. Fauve, J. Phys. II **6**, 1099 (1996).

[5] A. Pumir, Phys. Fluids **8**, 3112 (1996).

[6] P. Poggi and S. Fauve (unpublished).

[7] S. Bramwell, P. Holdsworth, and J.-F. Pinton, Nature (London) **396**, 552 (1998).

[8] P.J. Zandbergeeb and D. Djikstra, Annu. Rev. Fluid Mech. **19**, 465 (1987).

[9] J. Maurer, P. Tabeling, and G. Zocchi, Europhys. Lett. **26**, 31 (1994); J.-F. Pinton and R. Labbé, J. Phys. II **4**, 1461 (1994).

[10] A. Arnéodo *et al.*, Europhys. Lett. **34**, 411 (1996).

[11] We follow here a procedure that has been initiated for the detection of the pressure drops that sign the sweeping of vorticity filaments at the flow wall. S. Fauve, C. Laroche, and B. Castaing, J. Phys. II **3**, 271 (1993); O. Cadot, S. Douady, and Y. Couder, Phys. Fluids **7**, 630 (1995).

[12] General arguments are in H. Schlichting, *Boundary Layer Theory* (McGraw-Hill, New York, 1979); detailed studies in the von Kármán geometry are reported in O. Cadot, Y. Couder, A. Daerr, S. Douady, and A. Tsinober, Phys. Rev. E **56**, 427 (1997); N. Mordant, J.-F. Pinton, and F. Chillà, J. Phys. II **7**, 1729 (1997).

[13] K. Binder, Z. Phys. B **43**, 119 (1981).

[14] L. Landau and E. Lifchitz, *Fluid Mechanics* (Pergamon Press, New York, 1987), Chap. 33.

[15] A.J. Bray and M.A. Moore, J. Phys. C **18**, L927 (1985).